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Readers' Forum

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Comment on "Improvement in Model Reduction Schemes Using the System Equivalent Reduction Expansion Process"

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THE authors of Ref. 1 presented a numerical method for model reduction schemes using the system equivalent reduction expansion (SEREP) process. However, the method is valid only when the Guyan reduction has very high accuracy. It means that the validity of the method in Ref. 1 depends deeply on the choice of the master degrees of freedom (DOFs). A number of authors have considered the selection of an appropriate master DOF set.[2–4] Unfortunately, although the appropriate master DOF set is used, the errors of frequencies with Guyan reduction are usually very large.[5] Moreover, the selection schemes of master DOFs will be invalid in many cases.[6]

The numerical example of a uniform cantilevered beam[1] demonstrates our opinion. In Table 1, the master DOFs, equivalent to those of Ref. 1, are the transverse DOFs at nodes 2, 4, 6, 8, 10, and 12 for case 1. For case 2, the master DOFs are the transverse DOFs at nodes 7–12, and for case 3 they are the rotational DOFs at nodes 1–6.

The results indicate that when Guyan reduction has high accuracy, the SEREP–Guyan has smaller errors than the Guyan. However, when Guyan reduction has low accuracy, the SEREP–Guyan has larger errors than the Guyan at majority frequencies.

Table 1 Errors of beam natural frequencies (Hz) from different cases

	Case 1		Case 2		Case 3	
Mode	Guyan	SEREP–Guyan	Guyan	SEREP–Guyan	Guyan	SEREP–Guyan
1	0.000001	0.000000	0.000072	0.000000	0.025081	0.000056
2	0.000417	0.000000	0.026975	0.000012	0.311917	0.534706
3	0.003388	0.000000	0.335096	1.08737	0.917563	1.51624
4	0.014125	0.000023	0.946580	1.40021	1.50816	1.50714
5	0.037918	0.000387	1.67978	1.88315	2.15232	2.82557
6	0.040935	0.005863	2.51075	2.50444	2.92630	3.33190

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Reply to Z.-Q. Qu and Z.-F. Fu

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THE authors thank Qu and Fu for their interest and examination of our paper.[1]

They present a case where the SEREP–Guyan approach produces larger errors than the Guyan method for a different set of master

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Table 1 Beam natural frequencies in hertz for displacement DOF at nodes 1–6

Mode	Exact	Guyan	SEREP–Guyan	IRS	SEREP–IRS
1	16.1676	16.6414	16.1692	16.1676	16.1676
2	101.3223	136.8621	166.2888	101.3519	101.3277
3	283.7346	561.9052	604.4304	292.2192	286.3906
4	556.1992	1387.5427	1456.4366	687.7229	766.9553
5	920.1960	2723.6021	2763.6629	1551.4092	1617.2375
6	1376.7569	4761.4220	4667.9298	3248.3423	3296.4435

degrees of freedom (DOF). However, this should come as no surprise to anyone for the following observations. Upon closer examination, it is realized that our method closely parallels the classical Rayleigh method.[2] The reason is that the exact mode shapes are not used in the transformation matrix as required by SEREP. Rather, some assumed shape function was employed. In our case, the shape function was calculated by dynamically expanding the reduced mode shapes. Thus, the SEREP–Guyan method uses an assumed mode shape approach, whereas other methods improve upon the Guyan method through retaining higher-order terms[3] or accounting for missing inertia,[4] for example.

From a theoretical point of view, however, we admit the possibility of an alternative procedure for obtaining the full set of mode shapes other than through dynamic expansion. Baruch and Meirovitch[5] present a spline-based method where a smoothed modal shape is obtained from knowledge of a few points on a structure. This is exactly the case here. The Guyan procedure yields the mode shape information at a few select points, and it is desired to interpolate (or extrapolate, as it may be) to other nodal locations. One is not limited to using just the Guyan mode shapes; using a mode shape from any reduction procedure will work as well.

As pointedly noted by Qu and Fu, the drawback of our reduction procedure is that it is highly dependent on the accuracy of the reduced mode shapes. However, this restriction is overcome by the fact that, in the Rayleigh method, a first-order error in mode shape results in only a second-order error in frequency.[6] Should the assumed mode shapes, though, be grossly in error, it cannot be expected that the natural frequencies will be correct, which prompted us to make this statement in our paper: "The particular reduction scheme, however, is unimportant. The only underlying assumption is that the shape of each reduced mode resembles the full system mode, which inevitably is dependent on the choice of master DOF. Suffice it to say, good results are expected through a sufficient number and location of master DOF."[1]

The following is a case in point. Let us consider retaining the transverse DOF at nodes 1–6 for the same cantilevered Euler–Bernoulli beam example. The natural frequencies for the separate reduction methods are presented in Table 1. It is observed that only the first mode is reasonable when using Guyan. Consequently, the SEREP–Guyan method almost exactly identifies only the first mode. The other modal frequencies show considerable disagreement with the exact frequencies. The Improved Reduced System (IRS) method appears to have done a somewhat better job identifying up to the third mode. Consequently, the SEREP–IRS method shows much improvement for the first three modes. The remaining modes suffer larger errors.

Qu and Fu's case 2 consists of a questionable master DOF set because it will be difficult to identify the first six cantilever modes from that master set. Furthermore, the master DOF in case 3 are not considered reliable because they consist of rotations, and these DOF should be avoided or, at the least, set aside until all other displacement DOF have been exhausted. This was suggested from the work of Downs.[7] Most, if not all, reduction methods have difficulty when improper master DOF are selected.

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